

THE ANTI-FERROMAGNETIC VACUUM

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Certain effective vertices may generate a non-homogeneous, periodic vacuum structure. The excitations above such a vacuum are studied in the framework of the ϕ^4 and gauge models. The formation of the non-homogeneous vacuum is accompanied by the dynamical breakdown of the space-time inversion symmetry. Chiral transformation is introduced for bosons in close analogy with lattice fermions.

1 Introduction

The ferromagnetic condensate is a well known device for mass generation in particle physics ever since the seminal work of Nambu and Jona-Lasinio¹. This condensate is a coherent state of particles with vanishing momentum. We suggest in this talk the use of another type of condensate in Quantum Field Theory which is a coherent state consisting of particles with non-zero momentum. The field operator which has non-homogeneous vacuum expectation value in this vacuum oscillates with the characteristic momentum of the particles of the condensate. Based on the formal similarity with the Neél state of the anti-ferromagnetic Ising model this condensate will be called anti-ferromagnetic.

The realistic theories of Particle Physics are effective models since we do not know the physics up to infinite energies. Each effective theory has an energy range where it is supposed to be applicable and contains effective interactions due to the particle exchange processes occurring beyond its energy regime. The impact of these effective vertices is usually classified by the Appelquist-Carazzone decoupling theorem², which states that the non-renormalizable effective vertices are suppressed by the ratio of the light and the heavy particle mass. This result is usually taken as an indication that the effect of these vertices whose number is uncontrollably large is numerically small and it is enough to take into account the contributions of the heavy particle exchange processes to the renormalizable vertices only as far as the low energy physics is concerned.

It is overlooked in this argument that the decoupling theorem is based on

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a careful analysis of the loop corrections and one tacitly assumes that nothing important is going on at the tree level. Can it happen that the tree level semi-classical vacuum is modified by the heavy particle exchange contributions ? The dependence of the solution of the classical equation of motion on the non-renormalizable coupling constants is more complicated than those of the radiative corrections. The strategy for the classification of the impact of the effective vertices which is based on the decoupling theorem is not adequate to estimate the tree level effects of the heavy particle exchange processes.

Let us consider the strong and the electromagnetic interactions at finite baryon number density as an example. The heavy and light matter particles of this system are the nuclei and the electrons, respectively. For the low energy phenomena it is enough to consider the electrons and the slightly ionized atoms as matter particles. Imagine the effective theory for the electrons and the photons after the perturbative elimination of the ions. We know very well that the vacuum of this effective theory is the non-homogeneous solid state lattice, more precisely its electric field and the corresponding Dirac sea for certain baryon densities. This non-trivial vacuum can not be generated by the radiative corrections of the non-renormalizable effective vertices which are strongly suppressed according to the decoupling theorem. Instead, it is believed to be a semi-classical, tree level effect³, which is happened to be much more important than one would have expected according to the order of magnitude of the non-renormalizable effective vertices.

One of the usual strategies to "derive" the solid state lattice is based on the Born-Oppenheimer approximation where one eliminates the light electrons and describes the semi-classical motion of the heavy ions in the resulting non-local potential. It is more promising to eliminate the heavy degrees of freedom first and study the dynamics of the light particles in the presence of the effective local interactions later. We believe that the perturbative elimination of the ions will give us the effective theory whose semi-classical vacuum is the electric field of the solid state lattice.

This talk is devoted to some general remarks about the appearance of the non-homogeneous, anti-ferromagnetic vacuum in some models. We start with the simplest case, the ϕ^4 model in Section 2. The interpretation of the tree level phase transition is the subject of Section 3. The particle content is discussed in Section 4 by paying special attention to the space-time inversions. The generalization of the anti-ferromagnetic vacuum for gauge theories and some of its characteristic phenomena are discussed in Section 5. Finally Section 6 is for the conclusion.

2 Tree-level non-decoupling for the ϕ^4 model

Consider the Euclidean effective action

$$S[\phi(x)] = \int d^d x \left(\frac{1}{2} \partial_\mu \phi(x) \mathcal{K}(\partial^2) \partial_\mu \phi(x) + V(\phi(x)) \right) \quad (1)$$

for the scalar field $\phi(x)$ where the potential energy is symmetrical, $V(\phi) = V(-\phi)$. In order to arrive at a renormalizable theory we choose $V(\phi) = \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4$. The higher order, apparently non-renormalizable terms of the kinetic energy,

$$\mathcal{K}(z) = 1 + c_2 \frac{z}{\Lambda^2} + c_4 \frac{z^2}{\Lambda^4}, \quad (2)$$

are supposed to arise from some heavy particle exchange at the energy scale Λ . This energy scale plays the role of a smooth cut-off since the $O(p^{-6})$ propagator cuts off the loop integrals at $p = O(\Lambda)$. The coupling constants c_j are irrelevant as far as the quantum fluctuations around a homogeneous vacuum are concerned because they always appear with the suppressing factor $O((p/\Lambda)^j)$ in the radiative corrections.

In order to estimate the tree level effects of c_j we first suppose that the vacuum remains homogeneous, $\langle \phi(x) \rangle = \text{const}$. The quantum fluctuations, the eigenfunctions of the second functional derivative of the action are plane waves with the eigenvalue

$$\lambda(p^2) = m^2 + p^2 - c_2 \frac{p^4}{\Lambda^2} + c_4 \frac{p^6}{\Lambda^4}. \quad (3)$$

We shall consider $m^2 < 0$ in this talk, where one has a ferromagnetic condensate, $\langle \phi(x) \rangle = \text{const}$, for $c_j = 0$. But observe that for sufficiently large c_2 the eigenvalue becomes negative for non-vanishing momentum indicating an instability for particles with momentum $p = O(\Lambda)$. The coherent state of these particles generates a field expectation value which oscillates at the characteristic length $O(\Lambda^{-1})$. The numerical solution⁴ of the Euler-Lagrange equation of (1) supports this picture and results an oscillating classical field which is a pure cosinus up to 99% in $d = 1$ and a somewhat lower degree in higher dimensions. Such a small spread in the Fourier space indicates weak interaction between the particles which form the condensate.

For $|m^2| \ll \Lambda$, $3c_4 < c_2^2$, $\lambda(p^2)$ takes negative values and we have anti-ferromagnetic condensate. The ferromagnetic condensate appears when $3c_4 < c_2^2$ and $m^2 < 0$. We found no phase where both $p = 0$ and $p \neq 0$ condensates are present at the semiclassical level. The situation becomes more involved in lattice regularization. This is because there are in fact two lattices in this case,

one of those of the anti-ferromagnetism and the other is the regulator. One finds several regular phases which are labeled by the period length of the anti-ferromagnetic lattice measured in the units of the cut-off. These phases are separated by the incommensurable phases where the saddle point shows chaotic behavior.

3 Phase structure

The elementary excitations above the anti-ferromagnetic vacuum are the Bloch waves. The free inverse propagator (3) may have two minima along each axes in the momentum space so it is natural to introduce 2^d regions in the Brillouin zone $|p_\mu| \leq \pi/a$ around each saddle points of $\lambda(p^2)$, $p_\mu = P_\mu^\alpha(c_2, c_4)$, $\alpha = 1, \dots, 2^d$, which together cover the whole momentum space. If a saddle point is a local minimum then the excitations around it are particle like.

It is instructive to consider the theory in lattice regularization. The free inverse propagator is

$$G_0^{-1}(p) = a^2 m^2 + \mathcal{P}^2(p) - c_2 \frac{a^2}{\pi^2} (\mathcal{P}^2(p))^2 + c_4 \frac{a^4}{\pi^4} (\mathcal{P}^2(p))^3, \quad (4)$$

where $\mathcal{P}^2(p) = \frac{4}{a^2} \sum_\mu \sin^2 \frac{ap_\mu}{2}$. The key observation is that it is invariant under the transformation

$$p_\mu \longrightarrow p_\mu + P_\mu^{2^d}, \quad (5)$$

when

$$c_2 = \frac{1}{4d}, \quad c_4 = 0. \quad (6)$$

The vector P_μ^α is defined by $P_\mu^\alpha = P_\mu^\alpha \left(\frac{1}{4d}, 0 \right) = \frac{\pi}{a} n_\mu(\alpha)$, where the components of the vector $n_\mu(\alpha)$ is 0 or 1,

$$\alpha = 1 + \sum_{\mu=1}^d n_\mu(\alpha) 2^{\mu-1}. \quad (7)$$

Notice that the lattice action with $c_4 = 0$ is bounded from below due to the ϕ^4 term in the potential energy⁵.

The symmetry of the free propagator reflects the invariance of the kinetic energy with respect to (5). This transformation introduces a fluctuating sign for the field variable, $\phi(x) \rightarrow \pi\phi(x)$, where the operator π acts in the space of lattice field configurations as

$$\pi\phi(x) = (-1)^{\sum_{\mu=1}^d x^\mu} \phi(x) \quad (8)$$

and (5) leaves the potential energy invariant, too. We shall explore the consequence of this discrete symmetry of the theory (6) in two, slightly different manners.

The projection operator corresponding to the two sub-lattices of the anti-ferromagnetic order is $P_{\pm} = \frac{1}{2}(1 \pm \pi)$. For the symmetrical theory (6) we have $[\mathcal{K}(\partial^2)\partial^2, \pi] = 0$. Another important property we need is $\pi^2 = 1$. These two relations can be used to prove

$$P_- \mathcal{K}(\partial^2) \partial^2 P_+ = 0, \quad (9)$$

which shows that the field variables of the two sublattices are not coupled by the kinetic energy. Since the potential energy is ultralocal the two sublattices decouple and the theory (6) consists of two identical and independent scalar field theories defined on each sublattice.

Suppose that m^2 and g are chosen in such a manner that these two theories are in the symmetry broken phase. Then both of the decoupled fields develop non-vanishing vacuum expectation values which agree in their absolute magnitude and their sign is chosen randomly by a dynamical symmetry breaking mechanism. In the case when the two condensates have same (opposite) sign we find ferromagnetic (anti-ferromagnetic) vacuum in the space-time.

Let us consider the theory in the vicinity of the symmetric case⁸,

$$c_2 = \frac{1}{4d} + a^2 M^2, \quad c_4 = 0. \quad (10)$$

The theory is in the ferromagnetic or the anti-ferromagnetic vacuum for $M^2 < 0$ or $M^2 > 0$, respectively. The one-loop analysis⁴ shows that the ultraviolet divergences can be removed from the Green functions by the appropriate fine tuning of the bare parameters, $m^2(a)$, $g(a)$. The resulting renormalized theory contains two particles and its vacuum appears homogeneous for observations made at finite energies. The two particle species are non-degenerate for $M^2 > 0$ and as we shall show below the space-time inversion symmetry is dynamically broken. The cut-off independent finite energy physics belongs to different universality class than the usual ϕ^4 theory with $c_j = 0$. It remains to be seen if the triviality of the theory persists in this scaling regime.

The length scale of the condensate could be kept finite as in Solid State Physics. In this case the hard processes which tend to destroy the condensate reveal the presence of unitarity violating processes in the theory at high energies⁹. This is not a serious problem if the theory is used only to describe the effects of the low energy excitations. It is worthwhile noting that the classical solutions may contain ultraviolet Fourier modes, it is the excitation above the semi-classical, non-fluctuating vacuum what is supposed to be at

low energy in order the effective theory make sense. We have, for example, no doubt about the correctness of the description of the low energy elementary excitations in solids in terms of phonons and electrons. The hard photon exchanges which may induce lattice defects appear to violate the unitarity of the phonon-electron system and require the use of the complete, more fundamental QED treatment. This high energy violation of the unitarity disappears from the theory where the length scale of the condensate is renormalized to zero and we expect to find an acceptable theory without extra anomalies after the renormalization around the symmetric point⁴, (6).

4 Chiral bosons

A more systematical way of exploring the consequence of the symmetry (5) can be achieved by exploiting the formal similarities between our system and the lattice fermions. To this end we introduce the hyper-cube variables $x^\mu = 2y^\mu + n^\mu$ and the corresponding field⁶ $\phi_n(y) = \phi_\alpha(y) = \phi(2y + n(\alpha))$ where n and α will be called the helicity index. It is advantageous to introduce the linear superpositions⁷ $\tilde{\phi}_\alpha(y) = A_{\alpha\beta}\phi_\beta(y)$, where the matrix

$$A_{\alpha\beta} = 2^{-d/2}(-1)^{n(\alpha) \cdot n(\beta)} \quad (11)$$

represents the Fourier transformation on the helicity index. The Fourier transform of the field $\tilde{\phi}_\alpha(y)$ in y is non-vanishing for $|p_\mu| \leq \pi/2a$ and corresponds to the excitations in the restricted Brillouin zone around $p \approx P^\alpha$. Since and

$$(A^2)_{n,n'} = 2^{-d} \sum_m (-1)^{m \cdot (n+n')} = \delta_{n,n'}, \quad (12)$$

the inverse transformation is $\phi_\alpha(y) = A_{\alpha\beta}\tilde{\phi}_\beta(y)$.

The transformation (5) acts as $\tilde{\phi}_\alpha(y) \longrightarrow \tilde{\phi}_{2^d+1-\alpha}(y)$ on the Fourier transformed fields and is represented by $n \rightarrow E(0) - n(\text{mod}2)$ in the vector notation. The vector $E(k)$, $0 \leq k \leq d$ is defined here as

$$E_\mu(k) = \begin{cases} 1 & k \leq \mu, \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

One finds that the propagator has a saddle point in the restricted Brillouin zones of $\alpha = 2, \dots, 2^d - 1$ so the fields $\tilde{\phi}_\alpha(p)$ with these chiral indices do not correspond to particle like excitations⁴. On the contrary, the sectors $\alpha = 1$ and 2^d describe particle like excitations in the vicinity of (6) and the fields $\tilde{\phi}_1(y)$ and $\tilde{\phi}_{2^d}(y)$ which are mapped into each other by the symmetry transformation are the ferromagnetic and the staggered anti-ferromagnetic order parameters, respectively.

In order to demonstrate the geometrical origin of our discrete symmetry we discuss the inversions in space-time. The elementary cell of the anti-ferromagnetic system consists of the hypercube of 2^d points. The inversion I_μ of the coordinate x^μ flips the μ -th components of the chiral vector n_μ ,

$$I_\mu : \phi(y) \longrightarrow U_\mu \phi(I_\mu y). \quad (14)$$

The matrix U_μ defined in this manner acts on the helicity index,

$$(U_\mu)_{n,m} = \begin{cases} 1 & \text{if } n_\mu + m_\mu = \delta_{\mu,\nu}(\text{mod } 2), \\ 0 & \text{otherwise.} \end{cases} \quad (15)$$

The simultaneous inversion of all space coordinates, $P = \prod_{\ell=2}^d I_\ell$, induces the transformation

$$P : \phi(y) \longrightarrow U_P \phi(Py), \quad (16)$$

where

$$(U_P)_{n,m} = \left(\prod_{\ell=2}^d U_\ell \right)_{n,m} = \begin{cases} 1 & \text{if } n + m = E(1)(\text{mod } 2), \\ 0 & \text{otherwise.} \end{cases} \quad (17)$$

By analogy with the spin-half particles the field $U_P \phi(y)$ will be called the P-helicity partner of $\phi(y)$. The combined effect of the time inversion $T = I_1$ and the space inversion is represented by

$$PT : \phi(y) \longrightarrow U_{PT} \phi(PTy), \quad (18)$$

$$(U_{PT})_{n,m} = \left(\prod_{\mu} U_\mu \right)_{n,m} = \begin{cases} 1 & \text{if } n + m = E(0)(\text{mod } 2), \\ 0 & \text{otherwise.} \end{cases}, \quad (19)$$

and $U_{PT} \phi(y)$ will be called the PT-helicity partner of $\phi(y)$.

To make the analogy with the fermionic case more complete we shall determine the transformation rules for $\tilde{\phi}$ under space-time inversions. \tilde{U}_μ , the transformation matrix in the helicity space which represents I_μ is given by $\tilde{U}_\mu A = A U_\mu$, what yields $\tilde{U}_\mu = A U_\mu A$. By the help of

$$(\tilde{U}_P)_{n,n'} = (A U_P A)_{n,n'} = 2^{-d} \sum_m (-1)^{m \cdot (n+n') + n \cdot E(1)} \quad (20)$$

we find

$$(\tilde{U}_P)_{n,n'} = \delta_{n,n'} (-1)^{n \cdot E(1)} = \delta_{n,n'} (-1)^{\sum_{\ell=2}^d n_\ell}. \quad (21)$$

In a similar manner we have

$$(\tilde{U}_{PT})_{n,n'} = \delta_{n,n'} (-1)^{n \cdot E(0)} = \delta_{n,n'} (-1)^{\sum_{\mu=1}^d n_{\mu}}. \quad (22)$$

The formal similarity of this expression with (8) reveals that the space-time inversion, PT , is represented in the helicity space by the Fourier transform of our discrete symmetry (5). The distinct feature of the theory (6) is that the P- or the PT-helicity partners decouple for even or odd values of d , respectively. Such a theory will be called chiral invariant and its symmetry, (5), is a discrete version of a boose-chiral transformation. The continuous boose-chiral transformations can only be found in scalar field theories with continuous internal symmetry. The space-time inversions are diagonal on the chiral field basis of ϕ . The charge with respect to the exchange of the P-helicity partners, the P-chirality of ϕ_n , is defined as

$$\chi_P(n) = (-1)^{\sum_{\epsilon} n_{\epsilon}}. \quad (23)$$

The chiral transformation (5) which differs from that of the fermionic models by the presence of the time inversion is diagonal as well, its eigenvalue, the PT-chirality, is

$$\chi_{PT}(n) = (-1)^{n_0} \chi_P(n). \quad (24)$$

It is interesting to compare this situation with lattice fermions. The fermionic theory has 2^d non-interacting particle modes whose chiral charges sum up to zero. The different helicity partners decouple for the chiral invariant theory. The theory given by (1) displays a species doubling in each space-time direction. We introduced 2^d fields, $\phi_{\alpha}(y)$, which can be grouped into 2^{d-1} "biscalars", $\psi_{\alpha}^{PT\pm}(y) = \psi_{\alpha}(y) \pm U_{PT}\phi_{\alpha}(PTy)$, or $\psi_{\alpha}^{P\pm}(y) = \psi_{\alpha}(y) \pm U_P\phi_{\alpha}(Py)$. These pairs are classified by the help of an $O(2^{d-1})$ flavor group which acts on the helicity space. The members of the pairs are exchanged by the discrete transformation P or PT whose analogues are the Dirac matrices $U_P = i\gamma_0$ or $U_{PT} = \gamma_0\gamma_1\gamma_3$, respectively. The particle modes, $\tilde{\phi}_{\alpha}(p)$, which are the result of the diagonalization of the kinetic energy are the eigenvectors of the space-time inversions. The total chirality charge vanishes, $\sum_n \chi_P(n) = \sum_n \chi_{PT}(n) = 0$.

One may add further terms to the action in such a manner that the condensate occurs at $n = E(1)$. This is the Euclidean analogue of the solid state lattice where the vacuum is periodic in space but constant in time. Such a vacuum breaks the $O(d)$ Euclidean invariance and introduces a d -fold degeneracy in the spectrum. There is no species doubling in the time direction and $\chi_P(n) = \chi_{PT}(n)$ in this phase.

We note finally that the decoupling of the P-helicity members at (6) can be seen by repeating the argument used in the derivation of (9) but replacing the operator π by \tilde{U}_{PT} . The chiral invariance of the propagator implies $[\mathcal{K}(\partial^2)\partial_\mu^2, U_{PT}] = 0$. This relation together with $U_{PT}^2 = 1$ can be used to prove that the kinetic energy does not couple the chiral fields with opposite PT-chirality. The potential energy depends on $\phi_n^2(y) = \tilde{\phi}_m(y)A_{m,m'}^{(n)}\tilde{\phi}_{m'}(y)$ where

$$A_{m,m'}^{(n)} = (-1)^{n \cdot (m+m')}. \quad (25)$$

Due to $[A^{(n)}, U_{PT}] = 0$ any function of $\phi_n^2(y)$ leaves the opposite P-helicity partners decoupled.

5 Gauge theories

The phase structure of the simple SU(2) model with the extended action¹⁰

$$S = \sum_{x, \mu \neq \nu} \left(c_1 \nu \left[\text{square with } \mu \text{ and } \nu \text{ edges} \right] + c_2 \nu \left[\text{rectangle with } \mu \text{ and } \nu \text{ edges} \right] + c_3 \nu \left[\text{square with } \mu \text{ and } \nu \text{ edges} \right] \right) \quad (26)$$

is remarkable complex¹¹. This model has a number of phases which are characterized by different number of anti-ferromagnetic planes in which the plaquette shows staggered short range order. The fate of some characteristic features of QCD, such as asymptotic freedom, confinement, chiral symmetry breaking is unclear in the new phases. It was conjectured that the phase transitions can be used to construct a continuum theory with different universality class than for $c_2 = c_3 = 0$.

The anti-ferromagnetic long range order is characteristic of the semi-classical approximation. The quantum fluctuations may destroy the long range correlation leaving only a staggered short range order behind. This is supposed to happen if the theory is infrared unstable as in the case of asymptotically free models or spin liquids¹². The vacuum recovers the usual space-time translation symmetries after the long range order is removed.

The coupling of a matter field to this gauge model raises intriguing questions. The repetition of the construction of the chiral scalar fields outlined in the previous Sections indicates the possibility of constructing chiral gauge fields. Suppose that a complex scalar field is introduced with higher derivative terms in the action. In the anti-ferromagnetic phase of this model the helicity matter particle partners have different mass. In the low energy limit then one finds a chiral gauge theory for a single scalar chiral particle. Such a construction of a chiral gauge theory by the dynamical breakdown of the

space-time inversion symmetry is a reminiscent of the Higgs mechanism where the spontaneous breakdown of the internal symmetry is the key element in the construction of the desired gauge theory.

The dynamics is fundamentally influenced by the anti-ferromagnetic gauge field even if the matter field does not support the staggered order¹¹. One such a phenomenon is due to the one-loop effects of the matter particle. The periodic gauge field vacuum creates forbidden zones in the excitation spectrum in a manner analogous to the destructive Bragg reflection. Suppose that we have a massless fermion whose Dirac sea level is placed in the middle of such a gap by the help of a suitable chosen chemical potential. The gap opening in the excitation spectrum induces the dynamical breakdown of the chiral symmetry and leads to mass generation without Higgs particles. Another interesting phenomenon is related to the quantum fluctuations of the gauge field around the periodic vacuum. These are the analogues of the phonons of the solid state lattice. When the period length of the anti-ferromagnetic vacuum is renormalized to zero then the continuum translation symmetry is not restored in the continuum limit. So the phonons are not Goldstone modes and are not necessarily massless. When the length scale of the anti-ferromagnetic vacuum remains finite then the massless phonons introduce an unscreened attractive interaction between the fermions which may lead to the formation of the Cooper pairs and the superconductivity of the vacuum.

Finally we address the question whether the anti-ferromagnetic vacuum what has been discussed so far in models with freely adjustable coupling constants could actually be observed in realistic effective theories. Suppose that we eliminate n_f heavy fermions with mass M coupled to a gauge field. The resulting contribution to the effective action what can be derived by the help of the hopping parameter expansion in lattice regularization is the sum of Wilson loops. The coefficient of a Wilson loop Γ is $\beta_\Gamma = -n_f(-1)^{A(\Gamma)}O((aM)^{-L(\Gamma)})$, where $A(\Gamma)$ and $L(\Gamma)$ stand for the area and the perimeter of the Wilson loop in lattice units. The oscillating sign is the result of the fact that the product of the Dirac gamma matrices along a plaquette is -1. For an appropriate choice of n_f and M the semi-classical ground state can be made anti-ferromagnetic by the help of the negative coefficient of the double plaquette term.

The above argument applies to the lattice models with heavy masses but appears useless in continuum theories. The supercriticality¹³ of the QED vacuum for ions with $Z > C_{cr} \approx 173$ offers another non-perturbative mechanism without relying on the heavy mass expansion in lattice regularization. In fact, for sufficiently large amplitude oscillating electrostatic field the supercriticality occurs in each potential valley periodically. Since the electron states are emptied at the peaks of the potential and populated at the valleys the energy

of the resulting Dirac sea energy is lowered. Such a decrease of the fermion energy density is in a good approximation independent of the period length of the vacuum. The oscillating Dirac sea level "bends" the one particle wave functions and increases the Dirac sea energy. But this increase is small if the period length of the vacuum is long. The increase of the electrostatic energy density is proportional to the square of the wave vector of the vacuum. Thus for sufficiently strong and slowly varying external periodic electrostatic field the vacuum should be non-homogeneous¹⁴.

It is amusing to realize that a slightly similar vacuum based on the bag model has already been proposed¹⁵ for QCD. The dynamical origin of this picture of the vacuum is the conjecture that the short range perturbative modes generate an effective theory for the long range modes whose semi-classical vacuum is periodic. The non-perturbative aspects of the long range dynamics are supposed to take care of the homogeneity of the true vacuum, in a manner similar to the spin liquids.

6 Conclusion

The goal of this talk was to show the serious impact certain higher order derivative terms may have on the dynamics. If the scale of these higher order terms remains finite during the renormalization and we have an effective theory then phenomena, similar to Solid State Physics can be observed. If the length scale of the higher order terms can be removed together with the cut-off then qualitatively new continuum Quantum Field Theories are obtained containing new type of particles by the help of a dynamical symmetry breaking mechanism triggered by the kinetic energy.

The phenomena outlined in this talk are not yet well established, the renormalizability of the scalar field theory model is known up to one-loop only. Substantial work is left to achieve the level of accuracy which is usual for ferromagnetic models. But we believe that this new anti-ferromagnetic phase has sufficiently interesting features which are warrant of presentation even at this preliminary stage.

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